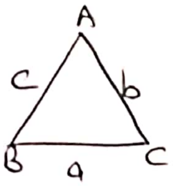
Heron's Formülü:

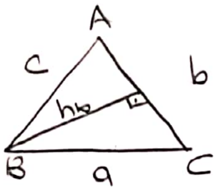


$s = \dfrac{a+b+c}{2}$ olmak üzere

$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$ dir.

Δ: Üçgenin alanı

İspat:



$\Delta = \frac{1}{2} b.c.\sin\hat{A}$ dır: Gerçekten

$\Delta = \dfrac{b.h_b}{2}$

$\sin\hat{A} = \dfrac{h_b}{c} \Rightarrow h_b = c\sin\hat{A}$

$\Rightarrow \Delta = \frac{1}{2} b.c.\sin\hat{A}$ olur.

$\Delta = \frac{1}{2} bc\sin A$

$\Delta^2 = \frac{1}{4} b^2c^2\sin^2 A = \frac{1}{4} b^2c^2(1-\cos^2 A)$

Ayrıca $a^2 = b^2+c^2-2bc\cos A$

$\Rightarrow \cos A = \dfrac{b^2+c^2-a^2}{2bc}$

$\Delta = \frac{1}{4} b^2c^2\left(1-\left(\dfrac{b^2+c^2-a^2}{2bc}\right)^2\right)$

$= \frac{1}{4} \cancel{b^2c^2}\left(\dfrac{4b^2c^2-(b^2+c^2-a^2)^2}{4\cancel{b^2c^2}}\right)$

$= \frac{1}{16}\left((2bc-b^2-c^2+a^2)(2bc+b^2+c^2-a^2)\right)$

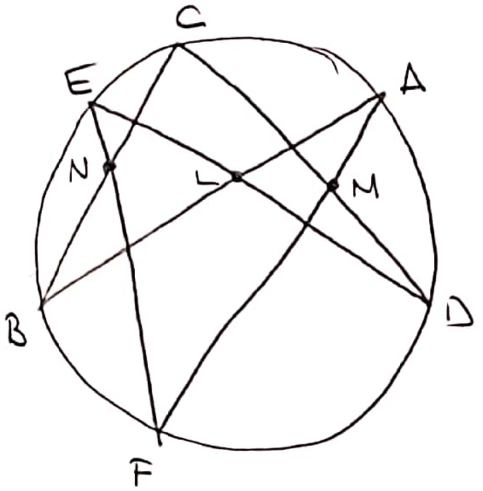
$= \frac{1}{16}\left(a^2-(b-c)^2\right)\left((b+c)^2-a^2\right)$

$\Delta^2 = \frac{1}{16}(a-b+c)(a+b-c)(b+c+a)(b+c-a)$

$= \dfrac{(a-b+c)}{2}\dfrac{(a+b-c)}{2}\dfrac{(a+b+c)}{2}\dfrac{(b+c-a)}{2}$

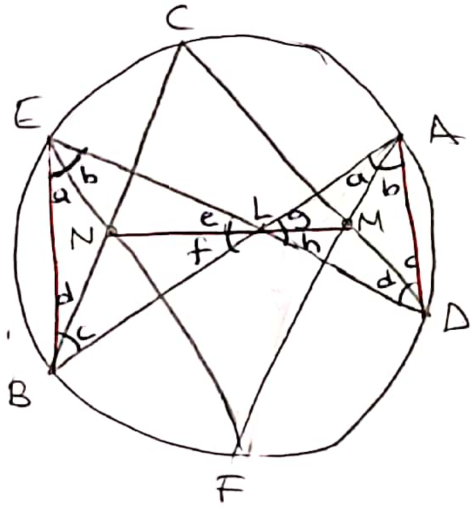
$= (s-b)(s-c)(s)(s-a)$

Pascal Teoremi!



N, L, M noktaları aynı doğru üzerindedir. (Doğrudaştır.)

İspat!



$m(E\hat{L}N) = m(M\hat{L}D)$

$e = h$ (Ters açı)

olduğunu göstermemiz yeterlidir.

$E\hat{L}B$ trigonometrik ceva'dan $\Rightarrow \sin a . \sin e . \sin c = \sin b . \sin f . \sin d$ --- (1)

$A\hat{L}D$ " " $\sin a . \sin h . \sin c = \sin b \sin g \sin d$ --- (2)

1 ve 2 oranlarsa

$$\frac{\sin e}{\sin h} = \frac{\sin f}{\sin g}$$

$\Rightarrow \sin e \sin g = \sin f . \sin h$ --- (3)

şekilden $e+f=g+h$ --(4) (ters açı, açık)

$\sin e.\sin g=\frac{1}{2}\left(\cos\left(\frac{e-g}{2}\right)-\cos\left(\frac{e+g}{2}\right)\right)$

$\sin h.\sin f=\frac{1}{2}\left(\cos\left(\frac{h-f}{2}\right)-\cos\left(\frac{h+f}{2}\right)\right)$

4) den $e+f=g+h \Rightarrow e-g=h-f$

3) den $\sin e\sin g=\sin h\sin f \Rightarrow \cos\left(\frac{e+g}{2}\right)=\cos\left(\frac{h+f}{2}\right)$

$\Rightarrow e+g=h+f$ -- (5)

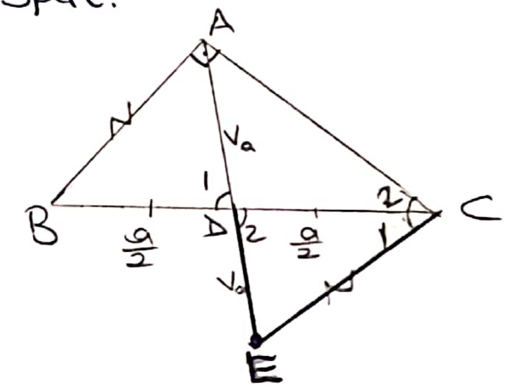
4 ve 5 den

$$\begin{array}{rl} & e-\cancel{g}=h-\cancel{f} \\ + & e+\cancel{g}=h+\cancel{f} \\ \hline & e=h \end{array}$$

$\rightarrow$ N, L, M doğrudaştır.

Teorem: Dik üçgende hipotenüse ait kenarortayın uzunluğu hipotenüsün yarısına eşittir.

İspat:



$m(\hat{A})=90°$

$|BD|=|DC|=\frac{a}{2}$

$V_a \overset{?}{=} \frac{a}{2}$

$|AD|=|DE|$ olacak şekilde bir E noktası alalım

$$\left.\begin{array}{l} |AD|=|DE| \\ |BD|=|DC| \\ m(\hat{D}_1)=m(\hat{D}_2) \end{array}\right\} \text{K.A.K} \Rightarrow \overset{\triangle}{ADB} \cong \overset{\triangle}{EDC}$$

$|AB|=|EC|$, $m(\hat{B})=m(\hat{C}_1)$

$\overset{\triangle}{ABC}$ dik üçgen old.dan

$m(\hat{B})+m(\hat{C}_2)=90°$ yazılabilir. $m(\hat{B})=m(\hat{C}_1)$ old.dan

$m(\hat{C}_1)+m(\hat{C}_2)=90°$ dir.

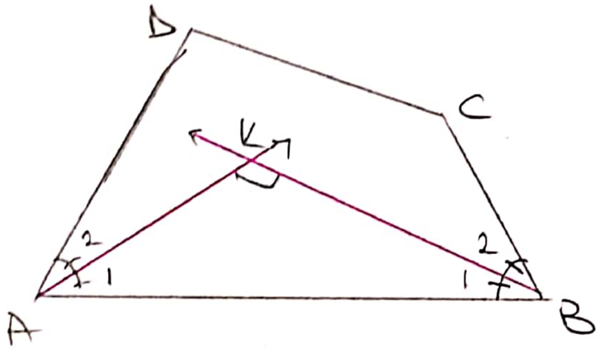
$\Rightarrow m(A\hat{C}E)=90°$

$$\left.\begin{array}{l} |AB|=|EC| \\ |AC|=|AC| \\ m(A\hat{C}E)=m(A) \end{array}\right\} \text{KAK dan } \overset{\triangle}{ABC} \cong \overset{\triangle}{CEA}$$

$\Rightarrow |AE|=|BC|$

$2V_a=|BC|=a \Rightarrow V_a=\frac{a}{2}$

Teorem: Bir dörtgende komşu iki köşedeki iki açının açıortayının oluşturduğu açının ölçüsü diğer iki köşedeki iç açıların toplamının yarısına eşittir.



$$m(A\hat{K}B) = \frac{1}{2}\left[m(\hat{C}) + m(\hat{D})\right]$$

İspat: $\overset{\triangle}{AKB}$ de

$$m(\hat{A_1}) + m(\hat{B_1}) + m(\hat{K}) = 180 \qquad m(\hat{A_1}) = \frac{m(\hat{A})}{2},\ m(\hat{B_1}) = \frac{m(\hat{B})}{2}$$

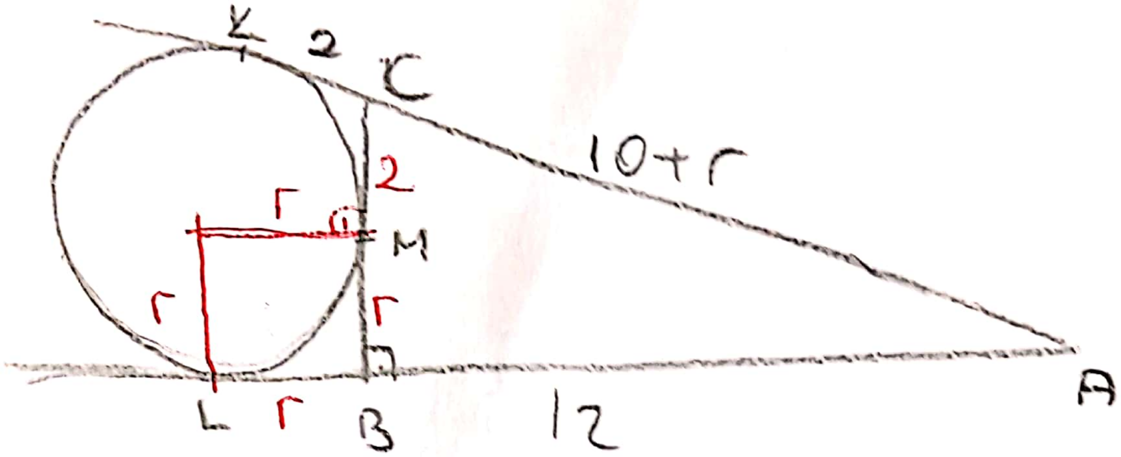
ABCD dörtgeninde

$$m(\hat{A}) + m(\hat{B}) + m(\hat{C}) + m(\hat{D}) = 360$$

$$\Rightarrow \underbrace{\frac{m(\hat{A})}{2} + \frac{m(\hat{B})}{2}}_{} + \frac{m(\hat{C})}{2} + \frac{m(\hat{D})}{2} = 180$$

$$180 - m(\hat{K}) + \frac{m(\hat{C}) + m(\hat{D})}{2} = 180$$

$$\Rightarrow m(\hat{K}) = \frac{1}{2}\left(m(\hat{C}) + m(\hat{D})\right)$$



ABC dik üçgeninden

$$(10+r)^2 = (2+r)^2 + 12^2$$

$$r = 3$$